Worker-Firm Dynamics with Seniority Bargaining

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joint with Miguel Portela[2] and Coen Teulings[3]

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  - economic rationale for ubiquitous *rents and bargaining power of insiders*

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  - wage regs on seniority; obs & spell-fixed unobs; identified apart from tenure (Topel, 91; Altonji & Shakotko, 87) or fsize effects (Brown & Medoff, 89)

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  - BT structural job spell parameters map into "cost of specific investment"

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  - facts: seniority-based promotion very used (Lazear & Oyer, 2012); wages reported discontinuous (Waldman, 2012)

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  - Show that *the interaction terms are fully driving the wage-seniority relationship*

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- worker outside wage (eg. return to self-employment), constant over time, normalized to unity, in logs $w^r = 0$

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$$\begin{aligned} p_t &= \pi, \\ n_t &= z_t - \eta \pi, \\ \pi &\equiv \ln \frac{\eta}{\eta - 1} > 0. \end{aligned}$$

Benchmark case: I=0

- labor demand adjusted costlessly at any time. Hence, subject to $n_t = z_t - \eta p_t$, maximize

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- constant firm price larger than log marginal cost $\pi > w = w^r = 0$, due to firm monopoly power; its labor demand follows random walk, i.e. Gibrat's law (Jovanovic, 1982)

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- optimal firm policy: hire whenever p_t reaches constant upper bound $p^+ > \pi$, fire whenever p_t reaches lower bound $p^- < \pi$ Picture.

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- ▸ Bargaining leads to log-linear sharing rule quasi-rents granting higher wages to inframarginal workers:

$$\begin{aligned} w(z_t - q) &= \beta \cdot mr\,(q, z_t) + \beta(\pi - p^-) + \omega \\ &= \frac{\beta}{\eta}\,(z_t - q) - \beta p^- + \omega \end{aligned}$$

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- β bargaining power of workers, ω log reservation wage of incumbent worker: $\beta = 1 \Rightarrow$ first degree price discrimination and full quasi-rents; $\beta = 0$ no worker bargaining power$\Rightarrow$ reservation wage.

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- ▸ Unemployment and welfare considerations.

Job spell distribution as first passage time

- Optimal hiring & separation for one worker-firm match reflected on z_t: hire at first time t that $z_t = z^S (= \eta p^-)$, fire at first time t that $z_t = z^T (= \eta p^+)$.

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- Distribution of completed spells Θ determined by the time it takes Ω_τ to pass barrier $\Omega_\tau = 0$ for first time, i.e. "First Passage Time" distribution

(cont)

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- Obtain cumulative distribution of jobs surviving at τ

$$\overline{F}(\tau) \equiv \Phi\left(\frac{\Omega + \pi\tau}{\sqrt{\tau}}\right) - e^{-2\Omega\pi}\Phi\left(\frac{-\Omega + \pi\tau}{\sqrt{\tau}}\right)$$

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  - US PSID: drift negative ▸ Picture; DK IDA & PT QdP drift positive.

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- Introduce Ω and π as tenure distribution parameters for each (4-digit, 3-digit) industry

Estimating structural job spell parameters

- Allow for observed, initial experience S, and unobs characteristics as random worker effects:

$$\Omega = \exp\left(\omega_0 + \omega_1 S + e_\Omega\right)$$
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where e_Ω and e_π normally distributed random effects with mean 0 and st dev σ_Ω and σ_π respectively

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- Ω and π estimated by SML, using the density function derived in the model. Simple log likelihood

$$\log L = \ln \int \int \prod_{j=1}^{J} \overline{F}(\Psi_j)^{1-d_j} \cdot f(\Theta_j)^{d_j} d\Phi\left(\frac{e_\Omega}{\sigma_\Omega}\right) d\Phi\left(\frac{e_\pi}{\sigma_\pi}\right)$$

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  - Estimate on 5% random sample of workers, per each industry; sensitivity checks.

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- $$\log w_{ijt} = \beta_0 + \beta_1 X_{ijt} + \beta_2 T_{ijt} + \beta_3 \log r_{ijt} + \beta_4 \log n_{jt} + \beta_5 Z_{ijt} + \varepsilon_{ijt}$$

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- Reasons φ_{ij}, ψ_j, and μ_i correlate to T_{ijt}, e.g., Altonji and Williams (2005)

(cont.)

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- Now *add in both regressions above (industry-specific) interaction terms* $r_{ijt} \times \Omega_{ind}$, and $r_{ijt} \times \pi_{ind} \times T$ and $\pi_{ind} \times T$

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  - Information on worker hourly earnings, education, age; firm's location, firm employment size, industry

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- DK vs. PT institutional framework; differences in EPL

Data Description (cont.)

	DK 1980-2001	DK 2000	PT 1986-2009	PT 2000
Variable				
Age	40.9	41.51	40.51	40.51
	(10.54)	(10.65)	(10.49)	(10.52)
Years of education	12.35	12.87	6.86	6.85
	(3.14)	(2.81)	(3.73)	(3.63)
Tenure	6.15	5.77	9.30	9.26
	(6.04)	(6.08)	(9.04)	(9.28)
Experience	22.93	23.41	24.04	24.05
	(11.14)	(10.73)	(10.81)	(10.85)
Log seniority	0.7	0.66	0.86	0.86
	(0.75)	(0.75)	(0.85)	(0.84)
Log firm size	4.7	4.77	4.14	4.05
	(2.33)	(2.35)	(2.11)	(2.14)
Log wage	3.15	3.2	1.52	1.61
	(0.3)	(0.32)	(0.53)	(0.52)
Sample size				
Observations	12634236	626867	15371019	725729
Workers	1412646	626867	2931323	725729
Firms	221807	60236	458888	124621
Spells	3456711	626867	4662627	725729

Notes: Standard deviations of variables in parentheses under their means. Log wages are expressed in euro and deflated to year 2000 prices.

Initial returns to seniority in Denmark (Buhai et al,2014)

	OLS		Topel		Altonji and Shakotko		Topel with spell fixed effects	
	I	II	I	II	I	II	I	II
Denmark								
$\log r_{ijt}$		0.0358		0.0079		0.0094		0.0102
		(0.0002)		(0.0003)		(0.0005)		(0.0004)
$\log n_{jt}$	0.0156	0.0155	0.0174	0.0124	0.0313	0.0258	0.0164	0.0094
	(0.0000)	(0.0000)	(0.0002)	(0.0003)	(0.0003)	(0.0005)	(0.0002)	(0.0004)
X_{ijt}	-0.0039	-0.0041	0.0478	0.0478	0.0316	0.0315	0.0284	0.0284
	(0.0002)	(0.0002)	(0.0004)	(0.0004)	(0.0003)	(0.0003)	(0.0003)	(0.0036)
T_{ijt}	0.0168	0.0068	-0.0060	-0.0082	0.0065	0.0040	0.0008	-0.0015
	(0.0002)	(0.0002)	(0.0009)	(0.0009)	(0.0001)	(0.0002)	(0.0062)	(0.0049)
T^2_{ijt}	-0.1736	-0.1004	0.1175	0.1360	-0.0245	-0.0081	0.1427	0.1592
	(0.0025)	(0.0025)	(0.0030)	(0.0033)	(0.0024)	(0.0025)	(0.0042)	(0.0042)
T^3_{ijt}	0.0744	0.0459	-0.0621	-0.0696	0.0075	0.0013	-0.0730	-0.0796
	(0.0014)	(0.0014)	(0.0018)	(0.0018)	(0.0013)	(0.0014)	(0.0022)	(0.0022)
T^4_{ijt}	-0.0099	-0.0062	0.0097	0.0108	-0.0003	0.0004	0.0113	0.0122
	(0.0002)	(0.0002)	(0.0003)	(0.0003)	(0.0002)	(0.0002)	(0.0004)	(0.0004)
R^2	0.0683	0.0710	0.0313	0.0313	0.2109	0.2211	0.0228	0.0229
Number of observations	12,275,995		8,597,167		12,275,995		8,597,167	

Initial returns to seniority in Portugal (Buhai et al, 2014)

	OLS		Topel		Altonji and Shakotko		Topel with spell fixed effects	
	I	II	I	II	I	II	I	II
Portugal								
$\log r_{ijt}$		0.0334		0.0150		0.0215		0.0043
		(0.0002)		(0.0004)		(0.0005)		(0.0011)
$\log n_{jt}$	0.0896	0.0884	0.0240	0.0144	0.0562	0.0434	0.0209	0.0185
	(0.0001)	(0.0001)	(0.0003)	(0.0004)	(0.0004)	(0.0005)	(0.0007)	(0.0010)
X_{ijt}	-0.1409	-0.1421	0.0704	0.0701	0.0613	0.0589	0.0729	0.0727
	(0.0004)	(0.0004)	(0.0007)	(0.0007)	(0.0006)	(0.0006)	(0.0015)	(0.0024)
T_{ijt}	0.0364	0.0312	0.0240	0.0218	0.0173	0.0149	0.0160	0.0156
	(0.0001)	(0.0001)	(0.0015)	(0.0014)	(0.0001)	(0.0002)	(0.0062)	(0.0066)
T^2_{ijt}	-0.2498	-0.2134	-0.0781	-0.0605	-0.0571	-0.0375	-0.1114	-0.1082
	(0.0014)	(0.0014)	(0.0022)	(0.0022)	(0.0015)	(0.0016)	(0.0035)	(0.0036)
T^3_{ijt}	0.0743	0.0617	0.0247	0.0188	0.0130	0.0083	0.0325	0.0315
	(0.0005)	(0.0005)	(0.0009)	(0.0009)	(0.0006)	(0.0006)	(0.0014)	(0.0014)
T^4_{ijt}	-0.0072	-0.0060	-0.0026	-0.0020	-0.0010	-0.0005	-0.0036	-0.0035
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0002)	(0.0002)
R^2	0.2385	0.2401	0.0333	0.0334	0.5071	0.5086	0.0116	0.0116
Number of observations	15,371,019		9,191,177		15,371,019		9,191,177	

Returns to seniority interactions with 1st stage parameters

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  - further legitimation for LIFO rule: not legal constraint, but efficient economic institution

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  - Back to "Theory Intuition"

Worker's optimization problem

- $V(z_t - q)$ asset value of holding a job at a firm

$$\rho V(z_t - q) = \exp\left[w(z_t - q)\right] + \mu V'(z_t - q) + \frac{1}{2}\sigma^2 V''(z_t - q)$$

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  - Smooth pasting: $V'(z_t - q) = 0$, for small variations in z_t, worker indifferent between holding the job & separating

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$$\omega = \ln r\left(\beta/\eta\right) - \ln\rho - \ln\left(1 - \frac{\beta}{\eta\lambda^{-}}\right)$$

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- $\frac{\partial\omega}{\partial\sigma} < 0$: declining in variability of demand σ^2, since higher variability raises option value of future surplus increases Back to "Wages and Seniority"

Firm's optimization problem

- $F(n_t - z_t)$ asset value of firm for N_t-th worker

$$\begin{aligned}\rho F(n_t - z_t) &= \exp[mr(z_t - n_t)] - \exp[w(z_t - n_t)] \\ &\quad + \mu F'(n_t - z_t) + \frac{1}{2}\sigma^2 F''(n_t - z_t)\end{aligned}$$

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- Suppose firm hires less than N_t workers. Then option value of hiring the Nth worker at some future date

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  - (iii) $\frac{\partial p^+}{\partial \beta} > 0$; (iv) $\frac{\partial p^-}{\partial \beta} < 0 \Longrightarrow$the higher β, the less volatile employment, as insulated over larger interval; Bertrand & Mullainathan (2003): firms insulated from takeover$\Rightarrow$wages of incumbents higher (higher β) ->lower rates of creation of new plants (higher p^+) and lower rate of destruction of old plants (lower p^-) Back to "Wages and Seniority"

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Back to "Wages and Seniority"

- log marginal revenue $mr(\cdot)$ ▸ Continue with "LIFO and rent sharing"

$$\ln\left[d\left(N_t \cdot P_t\right)/dN_t\right] \equiv mr\left(z_t - n_t\right) = \frac{1}{\eta}\left(z_t - n_t\right) - \pi$$

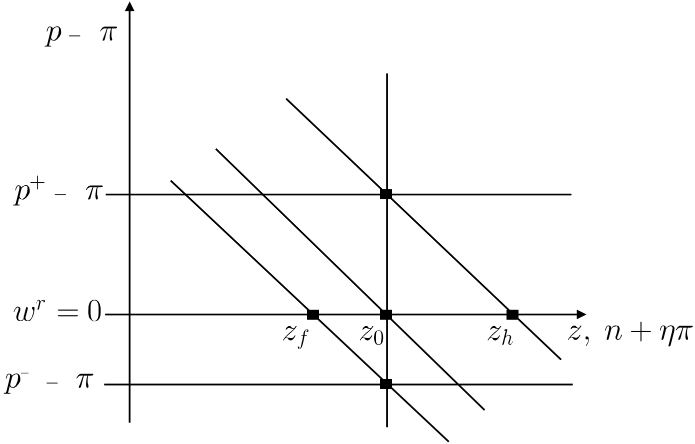

Figure: Firing-hiring boundaries with stochastic market index

Predicted job exit hazard

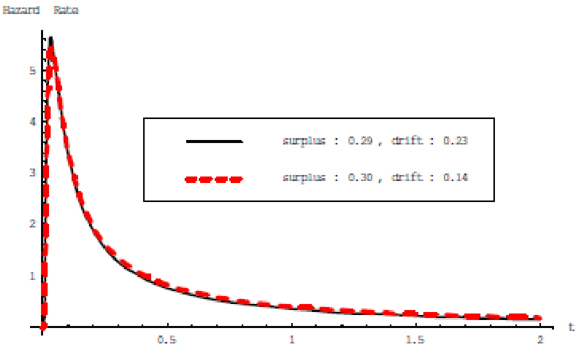

Figure 1: Predicted Job Hazards

Continue with "Definitions for empirical quantities"

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