# Ninth International Olympiad, 1967

### 1967/1.

Let ABCD be a parallelogram with side lengths AB = a, AD = 1, and with  $\angle BAD = \alpha$ . If  $\triangle ABD$  is acute, prove that the four circles of radius 1 with centers A, B, C, D cover the parallelogram if and only if

$$a \le \cos \alpha + \sqrt{3} \sin \alpha$$
.

#### 1967/2.

Prove that if one and only one edge of a tetrahedron is greater than 1, then its volume is  $\leq 1/8$ .

## 1967/3.

Let k, m, n be natural numbers such that m + k + 1 is a prime greater than n + 1. Let  $c_s = s(s + 1)$ . Prove that the product

$$(c_{m+1}-c_k)(c_{m+2}-c_k)\cdots(c_{m+n}-c_k)$$

is divisible by the product  $c_1c_2\cdots c_n$ .

## 1967/4.

Let  $A_0B_0C_0$  and  $A_1B_1C_1$  be any two acute-angled triangles. Consider all triangles ABC that are similar to  $\Delta A_1B_1C_1$  (so that vertices  $A_1, B_1, C_1$  correspond to vertices A, B, C, respectively) and circumscribed about triangle  $A_0B_0C_0$  (where  $A_0$  lies on  $BC, B_0$  on CA, and  $AC_0$  on AB). Of all such possible triangles, determine the one with maximum area, and construct it.

## 1967/5.

Consider the sequence  $\{c_n\}$ , where

$$c_{1} = a_{1} + a_{2} + \dots + a_{8}$$

$$c_{2} = a_{1}^{2} + a_{2}^{2} + \dots + a_{8}^{2}$$

$$\dots$$

$$c_{n} = a_{1}^{n} + a_{2}^{n} + \dots + a_{8}^{n}$$

$$\dots$$

in which  $a_1, a_2, \dots, a_8$  are real numbers not all equal to zero. Suppose that an infinite number of terms of the sequence  $\{c_n\}$  are equal to zero. Find all natural numbers n for which  $c_n = 0$ .

# 1967/6.

In a sports contest, there were m medals awarded on n successive days (n > 1). On the first day, one medal and 1/7 of the remaining m-1 medals were awarded. On the second day, two medals and 1/7 of the now remaining medals were awarded; and so on. On the n-th and last day, the remaining n medals were awarded. How many days did the contest last, and how many medals were awarded altogether?